Attenuation Law of Planar Shock Waves Propagating Through Dust-Gas Suspensions

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Introduction

A FEW years ago, Olim et al.[1] proposed a general law for describing the instantaneous shock wave Mach number as it attenuates while propagating through dust-gas suspensions. Their proposed attenuation law was

$$M_s = (M_0 - 1)\exp(-x/\chi) + 1 \tag{1}$$

where M_0 is the initial shock wave Mach number, i.e., the shock wave Mach number when it first encounters the dust-gas suspension whose edge is at $x = 0$. The attenuation coefficient χ was found to be linearly related to the diameter of the dust particles D and inversely to their loading ratio η through the expression

$$\chi = (1/\eta)[\alpha(M_0) + \beta(M_0)D] \tag{2}$$

In this expression α and β are constants for a given initial shock wave Mach number. Olim et al.[1] also calculated the appropriate values of these two constants for $M_0 = 1.5$. The appropriate values as obtained by them were $\alpha = 2.275$ m and $\beta = 3.07 \times 10^4$.

The correlation proposed by Olim et al.[1] implies that when $x \to \infty$ the instantaneous shock wave Mach number degenerates to a sound wave, i.e., $M \to 1$. This has been found experimentally[2,3] to be the case only if the initial shock wave is weak to moderate and the dust loading ratio is relatively high. If, however, the initial shock waves are strong and the dust loading ratios are low to moderate, the shock waves do not attenuate to sound waves but to finite strength shock waves whose Mach numbers M_e are larger than unity.[2,3]

Owing to this observation, it was suggested by Olim et al.[1] to replace Eq. (1) by the following more general expression:

$$M_s = (M_0 - M_e)\exp(-x/\chi) + M_e \tag{3}$$

Finding the functional dependence of both χ and M_e on the initial shock wave Mach number M_0, the diameter of the solid particles D, and the dust loading ratio η is the purpose of the study presented in the following sections.

Present Study

To find the functional dependence of both χ and M_e on M_0, D, and η, the governing equations describing the propagation of a planar shock wave through a dust-gas suspension, which were formulated by Olim et al.,[1] were solved numerically for various combinations of M_0, D, and η.

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Unlike Olim et al.[1] whose numerical simulation was based on the flux-corrected-transport technique (FCT),[4,5] the numerical simulation of the present study was based on the random choice method (RCM).[6,7]

The computer code was validated by comparing its predictions with all of the experimental results that were reported by Sommerfeld.[2] Good to very good agreement was evident. Details of the comparison can be found in Ref. 8, where a detailed derivation of the governing equations is also given. Consequently, in the following only the assumptions upon which the governing equations were based and their final form are presented.

Assumptions

1) The flowfield is one dimensional and unsteady.
2) The gaseous phase behaves as a perfect gas.
3) The solid particles are rigid, spherical, and inert. They are identical in all of their physical properties, including their diameter.
4) The solid particles are uniformly distributed in the suspension.
5) The number density of the solid particles is high enough to be considered as a pseudo gas.
6) The solid particles do not interact with each other, and as a result their partial pressure in the suspension is negligibly small.
7) The volume occupied by the solid particles is negligibly small.
8) Beside the momentum and energy exchange between the solid and the gaseous phases, the gaseous phase is assumed to be an ideal fluid, i.e., $\mu_g = 0$ and $k_g = 0$, where μ_g and k_g are the dynamic viscosity and the thermal conductivity, respectively.
9) The weight of the solid particles and the buoyancy force are negligibly small compared with the drag force acting on them.
10) The solid particles are too large to experience a Brownian motion.
11) The temperature within the solid particles is uniform.
12) The dynamic viscosity, the thermal conductivity, and the specific heat capacity at constant pressure of the gaseous phase depend solely on its temperature.
13) The specific heat capacity of the solid particles is constant.
14) The diameter of the solid particles is much larger than the shock wave thickness.

Governing Equations

Based on the previous assumptions the governing equations are the following:

Conservation of mass of the gaseous phase:

$$\frac{\partial \rho_g}{\partial t} + \frac{\partial(\rho_g u)}{\partial x} = 0 \tag{4}$$

Conservation of mass of the solid phase:

$$\frac{\partial \rho_p}{\partial t} + \frac{\partial(\rho_p v)}{\partial x} = 0 \tag{5}$$

Conservation of linear momentum of the gaseous phase:

$$\frac{\partial(\rho_g u)}{\partial t} + \frac{\partial\left(\rho_g u^2 + P\right)}{\partial x} = -F_D \tag{6}$$

Conservation of linear momentum of the solid phase:

$$\frac{\partial(\rho_p v)}{\partial t} + \frac{\partial\left(\rho_p v^2\right)}{\partial x} = F_D \tag{7}$$

Conservation of energy of the gaseous phase:

$$\frac{\partial}{\partial t}\left[\rho_g\left(C_v T_g + \frac{u^2}{2}\right)\right] + \frac{\partial}{\partial x}\left[u\rho_g\left(C_v T_g + \frac{u^2}{2} + \frac{P}{\rho_g}\right)\right] = -Q_{HT} - F_D v \tag{8}$$

Conservation of energy of the solid phase:

$$\frac{\partial}{\partial t}\left[\rho_p\left(C_m T_p + \frac{v^2}{2}\right)\right] + \frac{\partial}{\partial x}\left[v\rho_p\left(C_m T_p + \frac{v^2}{2}\right)\right] = Q_{HT} + F_D v \tag{9}$$

Equation of state of the gaseous phase:

$$P = \rho_g R T_g \tag{10}$$

In these equations, ρ_g, u, T_g, and P are the density, the velocity, the temperature, and the pressure of the gaseous phase, respectively; ρ_p, v, and T_p are the spatial density, the velocity, and the temperature of the solid phase, respectively. Note that since the solid phase does not have a pressure (see assumption 6), the pressure of the gaseous phase P is, in fact, the pressure of the entire suspension. The terms C_v and R are the specific heat capacity at constant volume and specific gas constant of the gaseous phase, and C_m is the specific heat capacity of the solid particles.

The previous set of seven governing equations [Eqs. (4–10)] contains seven dependent variables, namely, ρ_g, ρ_p, u, v, T_g, T_p, and P. Consequently, it could be solved provided the drag force per unit volume F_D and the heat transfer per unit volume Q_{HT} could be expressed in terms of these seven dependent variables.

The drag force per unit volume can be obtained from

$$F_D = \frac{3}{4}\frac{\rho_p \rho_g}{\sigma_p}\frac{C_D}{D}|u - v|(u - v) \tag{11}$$

where σ_p is the material density of the solid particles, D is their diameter, and C_D is the drag coefficient. The heat transfer per unit volume can be calculated from

$$Q_{HT} = 6\frac{\rho_p}{\sigma_p}\frac{h}{D}(T_g - T_p) \tag{12}$$

where h is the coefficient of convection heat transfer.

The drag coefficient C_D is usually correlated to the Reynolds number Re. Out of a variety of correlations proposed in Refs. 3 and 9–13, the one that was found to be most appropriate in reproducing all of the experimental results of Sommerfeld[2] is the one proposed by Clift et al.[9] It reads for $Re < 8 \times 10^2$

$$C_D = \frac{24}{Re}(1 + 0.15 Re^{0.687}) \tag{13a}$$

and for $Re > 8 \times 10^2$

$$C_D = \frac{24}{Re}(1 + 0.15 Re^{0.687}) + \frac{0.42}{1 + 42500 Re^{-1.16}} \tag{13b}$$

The Re number in these correlations is based on the relative velocity between the two phases, i.e.,

$$Re = \frac{\rho_g |u - v| D}{\mu_g} \tag{14}$$

where μ_g is the dynamic viscosity of the gaseous phase. The coefficient of convective heat transfer h appearing in Eq. (12) was calculated from

$$h = \frac{Nu k_g}{D} \tag{15}$$

where k_g is the thermal conductivity of the gaseous phase, and Nu, the Nusselt number, was calculated from the following correlation[14] that was found to be most appropriate in accurately reproducing all of the experimental results of Sommerfeld[2]:

$$Nu = 2 + 0.459 Pr^{0.33} Re^{0.55} \tag{16}$$

The Prandtl number Pr was calculated from

$$Pr = \frac{\mu_g C_p}{k_g} \tag{17}$$

where C_p is the specific heat capacity at constant pressure of the gaseous phase.

The governing equations [Eqs. (4–10)] were solved for the following ranges of parameters: $1 < M_0 \leq 5$, $25 \leq D \leq 500$ μm, and $0.25 \leq \eta \leq 10$. The upper limit of the initial shock wave Mach number value M_0 is due to assumption 2, namely, the gas behaves as a perfect gas. The diameter D is limited from above by assumptions 6, 7, 9, and 11, whereas its lower limit is governed by assumptions 10 and 14. The highest loading ratio value η is limited by assumption 7. To quantify the latter, the volume of the solid particles was limited to maximum 1% of the suspension volume. All of the 100 ($4 \times 5 \times 5$) possible combinations of the following values of the aforementioned three parameters were used as initial conditions for the numerical simulation undertaken during the course of this study: $D = 25, 50, 100$, and 500 μm; $\eta = 0.25, 0.75, 2.0, 6.0$, and 10.0; and $M_0 = 1.1, 1.5, 2.0, 4.0$, and 5.0.

As mentioned earlier, the purpose of the present study was to find the functional dependence of both χ and M_e, which are the free parameters in Eqs. (3) on M_0, D, and η. The process of evaluating this functional dependence is given in detail in Ref. 8. In the following only the final results are provided.

General Attenuation Law

For the reader's convenience the general attenuation law, which has been developed during the course of this study, is presented in the following:

$$M_s(x) = (M_0 - M_e)\exp\left(-\frac{x}{\chi}\right) + M_e \tag{18}$$

where the attenuation coefficient χ as a function of the parameters affecting the shock wave attenuation is given by

$$\chi(M_0, D, \eta) = \frac{b_1 D}{M_0}\eta^{-(b_2 M_0^{-b_3} + b_4)} \tag{19a}$$

and the equilibrium shock wave Mach number M_e, which is obtained at $x \to \infty$, is expressed by

$$M_e = A(M_0) + [M_0 - A(M_0)]\exp[-B(M_0)\eta] \tag{19b}$$

where

$$A(M_0) = b_5 + b_6 M_0 + b_7 M_0^2 \tag{20a}$$

and

$$B(M_0) = \frac{b_8}{\ln(b_9 M_0)} \tag{20b}$$

The values of $b_1, b_2, \ldots, b_9$ as obtained in the course of the present study and that ensure a 99% correlation of the curve fits with the numerical data are $b_1 = 0.02728$, $b_2 = 1.19194$, $b_3 = 6.63244$, $b_4 = 0.36737$, $b_5 = 1.15934$, $b_6 = -0.23607$, $b_7 = 0.10080$, $b_8 = 0.52311$, and $b_9 = 0.92796$. It could be seen that the general attenuation law given by Eq. (18) contains all of the flow parameters and dust physical properties that are known to affect the shock wave attenuation, namely, the initial shock wave Mach number M_0, the dust loading ratio η, and its diameter D.

It is evident from Eq. (19a) that the attenuation coefficient χ depends linearly on the particle diameter D and inversely on the dust loading ratio η. Consequently, $\partial\chi/\partial D > 0$ and $\partial\chi/\partial\eta < 0$. The dependence on the initial shock wave Mach number M_0 is more complicated.

The general attenuation law enables one to calculate the instantaneous shock wave Mach number within a 5% accuracy. As a result, the jump conditions across the attenuating shock wave could be readily obtained without the need to numerically solve the entire flowfield.

Validation of the General Attenuation Law

The shock wave attenuation, as predicted by the previously developed general attenuation law, was compared with the experimental results of Sommerfeld.[2,3] A typical comparison is shown in Fig. 1 for $M_0 = 1.7$, $D = 27$ μm, and $\eta = 1.4$. As can be seen, the agreement between the attenuation predicted by the general attenuation law [Eq. (18)] and the actual attenuation is very good.

Similar agreement was obtained when the predictions of the general attenuation law were compared with all of the experimental results of Sommerfeld.[2,3]

A comparison between the general attenuation law [Eq. (18)] and the attenuation law developed by Olim et al.[1] [Eq. (1)], which

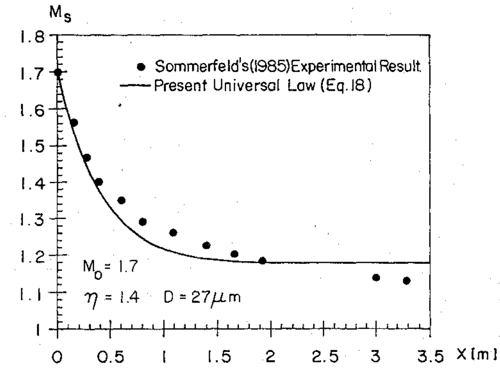

Fig. 1 Comparison between the attenuation predicted by the general attenuation law developed in the course of the present study and the experimental results of Sommerfeld.[3]

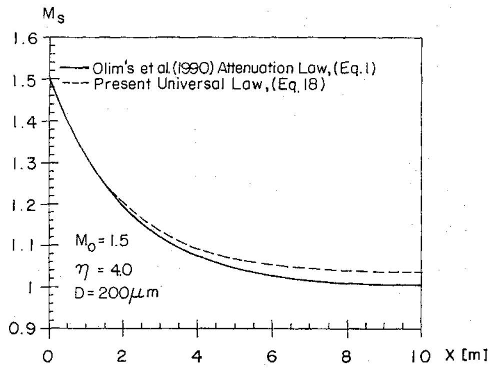

Fig. 2 Comparison between the predictions of the attenuation laws developed during the course of the present study (dashed line) and that developed by Olim et al.[1]

as mentioned earlier is applicable only to weak-to-moderate planar shock waves and relatively high loading ratios, is shown in Fig. 2 for $M_0 = 1.5$, $D = 200$ μm, and $\eta = 4.0$. It is evident that the general attenuation law as developed in the course of the present study predicts an attenuation similar to that predicted by Olim et al.'s[1] limited law. The difference of about 3 to 4%, which is developed at large values of x, is due to the fact that whereas the present general attenuation law dictates a realistic equilibrium shock wave Mach number, $M_e \geq 1$, Olim et al's[1] specific attenuation law inherently dictates $M_e = 1$ when $x \to \infty$.

Conclusions

The governing equations describing the flowfield that develops when a planar shock wave propagates inside a dust-gas suspension were solved numerically using the random choice method.

The shock wave attenuation was investigated numerically. Based on the numerical results, a general law for describing the shock wave attenuation as it propagates inside the dust-gas suspension was developed. It is an improvement of a simplified specific attenuation law that was developed a few years ago by Olim et al.[1] The results indicated that the coefficient of attenuation χ decreases as the dust particles diameter D decreases and the dust loading ratio η increases. As a result the attenuation itself increases when D decreases and η increases.

The general attenuation law was validated by comparing its prediction with actual experimental results. Very good agreement was evident.

The general attenuation law enables one to calculate simply and cheaply the instantaneous shock wave Mach number and as a result to obtain immediately the jump conditions across it without the need to conduct a tedious numerical simulation.

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Interaction of a Regular Reflection with a Compressive Wedge: Analytical Solution

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Introduction

An experimental and analytical study of the reflection processes of planar shock waves over double wedges was presented in Ref. 1. One out of the seven investigated double wedge combinations is shown in Fig. 1. The slopes of the first and second reflecting surfaces are θ_w^1 and θ_w^2, respectively, and θ_w^1 is large enough to ensure that the incident planar shock wave reflects over it as a regular reflection (RR). The case for which $\theta_w^2 < 90$ deg was investigated experimentally in Ref. 1 and numerically in Ref. 2, and the case

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